

DIFFRACTION EFFECTS IN MICROLENSING OF Q2237+0305

M. Jaroszyński^{1,2,3} and B. Paczyński^{1,4}

¹ Princeton University Observatory, Peyton Hall, Princeton, NJ 08544 USA

² Warsaw University Observatory, Al. Ujazdowskie 4, 00-478 Warsaw, Poland

³ E-Mail: mj@astrouw.edu.pl

⁴ E-Mail: bp@astro.princeton.edu

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ABSTRACT

Geometrical optics provides an excellent description for quasar images crossing caustics which are formed by gravitational microlensing of objects like Q2237+0305. Within this approximation the source size can be estimated from the maximum magnification reached at caustic crossings. We evaluate the limitations imposed by diffraction on caustics using the formalism developed by Ulmer & Goodman (1995). Close to a caustic a new characteristic length, smaller than the Fresnel length, enters the problem, limiting the angular resolution to about 0.2 pico arcsecond, or equivalently $\sim 3 \times 10^9$ cm at the source. To achieve this resolution the brightness must be monitored at time intervals of a few seconds. If a significant fraction of quasar luminosity comes from sources smaller than those limits then interference effects would make the observed intensity oscillate, in a close analogy with a two slit experiment. The characteristic period of such oscillations is expected to be about one tenth of a minute. If such oscillations are detected then photometry carried out at a single site may permit the determination of the caustic transverse velocity, and therefore may permit a direct conversion of the time units of brightness variations to the linear units at the source.

Subject headings: Gravitational lensing - dark matter - quasars: structure -quasars: Q2237-+0305

1. INTRODUCTION

The quadrupole gravitational lens 2237+0305 (Huchra et al. 1985) is the first, and so far the best studied case of gravitational microlensing at large optical depth. The four macroimages are formed by the whole galaxy acting as an astigmatic lens. The four lines of sight pass through four different inner regions of the lensing galaxy and are subject to microlensing by the stars located near those lines of sight. Some of the four macroimages were observed to vary on a time scales of months (Racine 1992 and references therein) and even days (Pen et al. 1994). The optical depth to macrolensing combined with the shear make the magnification of all four macroimages so large (cf. Wambsganss & Paczyński 1994, and references therein) that the microlensing is due to a collective effect of many stars, as emphasized by Wambsganss (1992). This is best seen at the magnification patterns, or equivalently the illumination patterns which clearly demonstrate that high magnification events are mostly caused by the source crossing one of the many caustics created by the microlensing, and almost none is caused by an isolated star (Wambsganss, Paczyński & Schneider 1990).

The generic structure of almost every caustic crossing is the following. On one side of the caustic the total magnification of a point source varies as $A = A_{\min} + (d_c/r_s)^{1/2}$ (cf. Fig. 1), where A_{\min} is the magnification on the other side of the caustic, r_s is the linear distance between the source and the caustic, and d_c is a characteristic length related to the caustic. In this paper all lengths, distances, and velocities in directions perpendicular to the line of sight are measured in the lens plane, while their equivalents in the source plane are denoted by the same symbols with hats. The length scale d_c is proportional to the Einstein ring radius r_E of individual microlenses, the stars in the lensing galaxy. If the source moves with a velocity V at an angle θ to the caustic then the magnification varies with time as

$$A \approx A_{\min} + (d_c/\Delta t V \sin \theta)^{1/2} = A_{\min} + (\Delta t_c/\Delta t)^{1/2}, \quad (1a)$$

$$\Delta t_c = d_c/V \sin \theta = \alpha r_E/V \sin \theta \sim M^{1/2}/V \sin \theta, \quad \alpha \equiv d_c/r_E, \quad (1b)$$

where Δt_c is the characteristic time related to the caustic, M is the mass of microlensing stars, and α is a dimensionless parameter of the order unity. The magnification of a point source increases all the way to infinity upon crossing any caustic. When the caustic is crossed the image magnification drops discontinuously to a finite value A_{\min} . Naturally, the crossing is equally likely to be in the opposite direction, with a discontinuous increase of the image magnification from A_{\min} to infinity, followed by a gradual decline described with the equations (1a,b).

If the source has a finite size, d_s , then the maximum magnification at the caustic

crossing is

$$A_{\max} \approx A_{\min} + (d_c/d_s)^{1/2}, \quad (2)$$

and the discontinuous jump in the magnification is replaced by a change on a time scale

$$\Delta t_s \approx d_s/V \sin \theta, \quad (3)$$

where Δt_s is the characteristic time related to the source.

Note that the high magnification events caused by caustic crossings are characterized by two distinctly different time scales. One time scale, Δt_c is proportional to the square root of some average mass of the stars that are responsible for microlensing. The other time scale, Δt_s is proportional to the source size. With frequent and accurate photometric measurements it should be possible to partly deconvolve the source structure (Grieger, Kayser, & Refsdal 1988; Wambsganss & Paczyński 1991).

Pen et al. (1994) presented the first observations of 2237+0305 variability which were interpreted as caused by a caustic crossing. The image A increased its brightness by ~ 1.5 magnitude over two months, corresponding to $\Delta t_c \approx 3 \times 10^6$ s, and subsequently its brightness dropped by ~ 1.5 magnitude in less than a week, corresponding to $\Delta t_s \approx 3 \times 10^5$ s. If our transverse velocity with respect to the lensing galaxy at $z_g = 0.04$ is $\sim 10^3$ km s $^{-1}$, then the projected velocity at the source at $z_s = 1.7$ is $\hat{V} \sim 10^4$ km s $^{-1}$, and the source size may be estimated to be $\hat{d}_s \sim \hat{V} \theta \Delta t_s \sim 3 \times 10^{14}$ cm. This is just ~ 20 astronomical units, much less than the estimates based on accretion disk models: $d_s \approx 1.5 \times 10^{16}$ cm (Rauch & Blandford 1991) or $\hat{d}_s \approx 4 \times 10^{15}$ cm (Jaroszyński, Wambsganss, & Paczyński 1992, cf. also Czerny, Jaroszyński, & Czerny 1994; Jaroszyński & Marck 1994; Witt & Mao 1994).

There is a possible way to reconcile the results of Pen et al. (1994) with the standard disk model of the quasar 2237+0305 if the event was caused by a very fast moving caustic crossing the image of the quasar. Such fast caustics were found by Kundić & Wambsganss (1993) and by Kundić, Witt & Chang (1993) to be the consequence of random motion of the stars responsible for microlensing. Only future observations and the well sampled light curves are likely to resolve this issue.

In the geometrical optics approximation the caustics are infinitely sharp and in principle the resolution of quasar structure is limited only by the accuracy and frequency of photometric measurements during the rapid change of the apparent brightness caused by a caustic crossing. However, at some level diffraction effects must limit the resolving power of any microlensing system. The aim of this paper is to determine what is the highest resolution that might be achieved. In the next section we treat the problem theoretically and in Sec.3 we apply our calculations to Q2237+0305. Discussion of the observability of the effects follows in the last section.

2. DIFFRACTION BY CAUSTICS

The diffraction effects near optical caustic have the same nature as in a two-slit experiment: they are caused by the interference of each photon passing the two paths corresponding to the two images which appear or disappear when the source crosses the caustic. The interference of radiation coming to an observer along different paths through the gravitational field, has been investigated by several authors. (Schneider & Schmidt-Burgk 1985; Deguchi & Watson 1986; Peterson & Falk 1991; Gould 1992; Stanek, Paczyński & Goodman 1993; Ulmer & Goodman 1995). In most cases the calculations were limited to the simplest case of a single point-mass lens. The effects of physical optics in gravitational lensing are referred to as femtolensing (Gould 1992).

Recently Ulmer & Goodman (1995, hereafter UG) have presented the method of calculating femtolensing effects for a general gravitational lens system. In particular they obtained the solution for the case of a source close to a caustic. In this paper we estimate the importance of the femtolensing for quasars. We are interested in the effects detectable with a broad band photometry, rather than the femtolensing effects in the spectra.

We use a general expression for the time delay along a ray crossing the lens surface at point $\mathbf{r} \equiv (x, y)$ and coming from a source point $\mathbf{r}_s \equiv (x_s, y_s)$ as projected onto the lens plane (cf. Blandford & Narayan 1986):

$$c\tau(x, y, x_s, y_s) = \frac{1+z_g}{2D} ((x-x_s)^2 + (y-y_s)^2) + (1+z_g)c\tau_g(x, y) \quad (4)$$

where τ is the total time delay and τ_g is the delay caused by the gravitational field of the lensing galaxy, $D \equiv D_g D_{gs}/D_s$ (in standard notation) is the characteristic gravitational lens distance and z_g is the redshift of the lens. Suppose we choose our coordinate system in such a way, that locally, near the origin, the line $x = 0$ is tangent to a critical line related to a fold caustic. In the simplest case we have $\tau_{,xx} = 0$ on the critical line $x = 0$. For a system consisting of point lenses one has $\tau_{g,xx} + \tau_{g,yy} \equiv 0$ everywhere, except singular points located at point masses. This equation fixes the value of the $\tau_{,yy}$ derivative on the critical line, where $\tau_{,xx} = 0$. Introducing ψ - a quantity proportional to the time delay and expanding it in the lowest nontrivial order we have:

$$\psi \equiv \frac{Dc\tau}{1+z_g} = \frac{1}{6}ax^3 + y^2 - x_sx - y_sy + \frac{1}{2}x_s^2 + \frac{1}{2}y_s^2 \quad (5)$$

where a is defined by the third x-derivative of the time delay and can be made positive by the transformation $x \rightarrow -x$. There are other parametrizations possible near a critical line (i.e. Schneider & Weiss 1986; Kayser & Witt 1989; Witt 1990; Witt, Kayser, & Refsdal 1993). The caustic line in our case is locally given by $x_s = 0$. Using equation (1a) we have

near the caustic:

$$A - A_{\min} = \left(\frac{d_c}{r_s} \right)^{1/2}, \quad (6)$$

where r_s is a distance of the source from the caustic line and d_c is a parameter characterizing the caustic. In our case $r_s = x_s$. The magnification due to the caustic related images is proportional to the inverse of the $||\psi_{,ij}||$ determinant, where ψ is given by equation (5). Comparing with the above formula we get:

$$d_c = \frac{1}{8a} \quad (7)$$

and substituting into the expression for ψ we have:

$$\psi = \frac{1}{48d_c} x^3 + y^2 - x_s x - y_s y + \frac{1}{2} x_s^2 + \frac{1}{2} y_s^2. \quad (8)$$

Physical optics calculations introduce the Fresnel length, which in our case can be defined as:

$$d_F = \left(\frac{\lambda D}{2\pi} \right)^{1/2} \equiv \left(\frac{cD}{\omega} \right)^{1/2}, \quad (9)$$

where λ is the wavelength and ω the frequency of light. With this definition one can calculate the complex amplitude (UG):

$$\Psi(x_s, y_s) = \frac{-i}{2\pi d_F^2} \int_{-\infty}^{+\infty} dx \int_{-\infty}^{+\infty} dy \exp\left(\frac{i\psi}{d_F^2}\right), \quad (10)$$

The normalization is such, that in the absence of the lens $\Psi(x_s, y_s) = 1$. In the case of a point source near the caustic line we get:

$$|\Psi(x_s, y_s)| = 2^{1/3} \pi^{1/2} \left(\frac{d_c}{d_F} \right)^{1/3} |\text{Ai}(-r_s/d_f)|, \quad (11)$$

where Ai is the Airy function (Abramovitz & Stegun 1964) and the characteristic scale for its argument changes is:

$$d_f = \frac{d_F}{2^{4/3}} \left(\frac{d_F}{d_c} \right)^{1/3}. \quad (12)$$

Both results are in agreement with UG but we use different parametrization. Using asymptotic expansion of the Airy function one can see, that for $r_s \gg d_f$ the flux from a point source $F(r_s) = |\Psi|^2 \rightarrow (d_c/r_s)^{1/2}$ as in geometrical optics calculation.

We have checked the sensitivity of the interference effects to the size of the source and the width of the frequency filter used for observations. For a monochromatic incoherent source with surface brightness $I(x_s, y_s)$ one finds its flux by the convolution:

$$F \sim \int dx_s \int dy_s I(x_s, y_s) |\Psi(x_s, y_s)|^2 \quad (13)$$

In Figure 2 we show light curves of monochromatic sources crossing the caustic. All sources are of the gaussian shape with characteristic sizes of 0.1, 0.3, 1, 3 and $10 \times d_f$. For sources with sizes $d_s < d_f$ the interference pattern is clearly visible and present at any distance from the caustic. The asymptotic expansion of Airy function (Abramovitz & Stegun 1964) $\text{Ai}(-x) \sim \sin(x^{3/2} + \pi/4)$ where we neglected the slowly changing amplitude, shows, that the spatial distance between consecutive fringes is

$$\Delta r_s = \frac{2\pi}{3} d_f \left(\frac{d_f}{r_s} \right)^{1/2} \quad (14)$$

The spacing Δr_s is proportional to the smoothed magnification, $A \sim (d_f/r_s)^{1/2}$. For a source moving with the velocity V as measured in projection onto the lens plane and perpendicular to the caustic, the characteristic time for time variations is given as

$$\Delta t \equiv \frac{\Delta r_s}{V} \quad (15)$$

This spatial and temporal characteristics of interference fringes is a generic feature of a motion across any caustic.

Let us consider now a typical photometric measurement using a broad band filter with a band pass $\Delta\omega/\omega = 0.2$. We assume the source spectrum to be flat in the filter range and use the "top hat" filter. We make convolution in frequency as well as in space. (Amplitude Ψ depends on frequency through d_F and d_f). The results are shown on Figure 3 for the same shapes of sources as previously. Only close to the caustic (where the phase of Ψ is the same for all frequencies) the interference pattern can be noticed. Farther away the fringes disappear. The number of clear fringes is $\sim \omega/\Delta\omega$.

3. APPLICATION

The caustic patterns were modeled by Wambsganss (1990), Witt (1990), Wambsganss, Witt, & Schneider (1992), and Witt et al. (1993, hereafter WKR) among others. The last of the quoted papers can be applied directly to our problem, since it gives the useful characteristics of caustics in the lensing system of Q2237+0305. We use the image A characteristics in our parameter estimation. The dependence of source magnification near caustic on the average microlens mass given by WKR shows that the relevant scale is the Einstein radius calculated for the average microlens mass:

$$r_E = \left(\frac{4GM_{av}D}{c^2} \right)^{1/2} = 1.6 \times 10^{16} \text{cm} (M_{av}/M_\odot)^{1/2} h_{75}^{-1/2}, \quad (16)$$

where h_{75} is the Hubble constant in units of $75 \text{ km s}^{-1} \text{ Mpc}^{-1}$. Now using WKR Table 4 and translating their notation to ours we get the value of $\alpha \equiv \langle K^2 \rangle / \langle m \rangle$ where K is their

flux factor and $\langle m \rangle$ is the averaged microlens mass in solar units. For image A we get $\alpha = 0.8$. The value of α may be somewhat different for other macro-image parameters. Substituting we have:

$$d_c = \alpha r_E = 1.28 \times 10^{16} \text{cm} (M_{\text{av}}/M_{\odot})^{1/2} h_{75}^{-1/2} . \quad (17)$$

The Fresnel length for an optical wavelength calculated for the same system is

$$d_F = 6 \times 10^{10} \text{cm} \lambda_{5000}^{1/2} h_{75}^{-1/2} , \quad (18)$$

where λ_{5000} is the wavelength in units of 5000 Å. Using this estimates of the characteristic scales we find that the limiting size for a source to show the diffraction effects at optical wavelength is:

$$\hat{d}_s < \hat{d}_f = \frac{D_s}{D_g} d_f \approx 3 \times 10^9 \text{cm} (M_{\text{av}}/M_{\odot})^{-1/6} \lambda_{5000}^{2/3} h_{75}^{-1/2} , \quad (19)$$

This is the upper limit to the source size as measured in the source plane, which for Q2237+0305 is at 8 times the distance to the lens plane. Also, this is the best resolution of a quasar that can be achieved through gravitational microlensing.

A characteristic time of femtolensing variability (assuming the relative velocity of 600km s⁻¹ at the lens plane which translates to 5000km s⁻¹ at the source plane, values used in Q2237+0305 modeling) is

$$\Delta t \approx 6 \text{ s} (M_{\text{av}}/M_{\odot})^{-1/6} \lambda_{5000}^{2/3} h_{75}^{-1/2} . \quad (20)$$

4. DISCUSSION

In our calculations we neglected the flux coming to an observer from images not related to the critical line. This is justified as the source crossing of a caustic does not influence other images and the pair of merging images dominates the total flux.

The diffraction limits the resolution achievable through photometric monitoring of caustic crossing to $\hat{d}_f \approx 3 \times 10^9 \text{cm} (M_{\text{av}}/M_{\odot})^{-1/6} \lambda_{5000}^{2/3} h_{75}^{-1/2}$ where M_{av} is the average mass of microlensing objects in the galaxy that macrolenses Q2237+0305. This corresponds to the angular resolution of $\sim \hat{d}_f/D_s$, where D_s is the angular diameter distance to the source, i.e. the resolving power of a caustic crossing in the Huchra's lens is $\sim 10^{-18}$ radians or 0.2 pico arcseconds in optical light for stellar mass microlenses. This is a very impressive resolving power indeed.

If there is a substructure in the Q2237+0305 with a scale smaller than \hat{d}_f then interference pattern should be detectable photometrically during the caustic crossing events:

a series of luminosity fluctuations with increasing or decreasing amplitude with the characteristic time scale of few seconds (cf. eq. [20]). Notice that if such fluctuations were detected they would allow the determination of the transverse velocity of the caustic, or equivalently they would allow the determination of a relation between the time scale of light variability and the linear size at the source using single site observation. If no interference effects are detected the determination of the caustic velocity requires at least two observing sites separated by about one astronomical unit, as first pointed out by Grieger et al (1988).

Our estimates demonstrate that in the case of Q2237+0305, and similarly other quasars, the geometrical optics is a very accurate approach. Investigating light curves of quasars crossing caustics one can probe their spatial structure down to scale as small as $0.05R_{\odot}$. Currently we have no reason to expect that any quasar has structure as small as a tenth of the solar radius, but it is good to know that structure down to such a small scale can be resolved with gravitational microlensing. It is also important to realize that the variation of image brightness should be monitored on the shortest time scales allowed by current technology in order to determine observationally what is the highest magnification present at caustic crossings, and what is the shortest time scale on which the brightness varies.

Adopting the geometric optics approximation we can estimate the source size d_s combining the equations (1b) and (2):

$$d_s = \frac{\alpha}{(A_{\max} - A_{\min})^2} r_E, \quad (21)$$

where $\alpha \approx 1$ (cf. Table 4 of WKR), r_E is the Einstein ring radius of the microlensing masses, A_{\max} is the maximum magnification reached at the caustic crossing, and A_{\min} is the magnification just outside the caustic. Notice, that the estimate does not require any knowledge of either the transverse velocity V , or the angle θ (cf. eq. [1a]).

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FIGURE CAPTIONS

- Fig.1. A schematic representation of a generic light curve when a point source crosses a caustic, as expected within a geometric optics approximation.
- Fig.2. Caustic-related magnification for gaussian shaped, monochromatic sources as a function of their distance from the caustic. The plots are for the sources with the characteristic size of 0.1 (the thinnest solid line), 0.3, 1, 3 and $10 \times d_f$ (the thickest line), where d_f is characteristic length near the caustic (eq. [12]). The geometrical optics result for a point source (eq. [6]) is shown for comparison as a dotted line. We adopt parameters of Q2237+0305 lensing system (eqs. [17,18]) for $M_{av} = 1 M_\odot$ and $\lambda_{5000} = 1$ to get the magnification values near the caustic.
- Fig.3. The same as on Fig.2 but for sources seen through a broad filter $\Delta\omega/\omega = 0.2$.